DOI:10.26104/IVK.2019.45.557

ИЗВЕСТИЯ ВУЗОВ КЫРГЫЗСТАНА, № 8, 2019

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ЭКСПЕРИМЕНТАЛДЫК МААЛЫМАТТАРДЫН САНДЫК НАТЫЙЖАЛАРЫН ИШТЕП ЧЫГУУ

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ЧИСЛЕННАЯ ОБРАБОТКА РЕЗУЛЬТАТОВ ЭКСПЕРИМЕНТАЛЬНЫХ ДАННЫХ

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NUMERICAL PROCESSING OF EXPERIMENTAL DATA RESULTS

УДК: 519.65

Берилген иштеги айкын проблемаларга жана прикладдык маселелерге байланыштуу түшүндүрмө берилген функцияларды чечмелөөнүн, кокустан түзүлгөн маалыматтарды текшилөөнүн милдеттери каралган. Бул маселелер жуурулушуу маалымат түрүндө дем алыш маалыматтарды берүү менен автоматташтырылган долбоорлоо системалары класстан математикалык эсептөөлөр боюнча MathCAD интегралдык программалык камсыздоону колдонуу менен чечилди. Математикалык пакеттер билдирген өз ара ыңгайлуу жана көп функционалдуу колдонмо программаларды ишке ашырууга жардам сандык эксперименттер, чечүүгө көп сандагы аппроксимационных милдеттерди. MathCAD пакеттин талашсыз функционалдык мүмкүнчүлүктөрү куралдар менен камсыз кылат, математикалык моделдөө кеңири адистештирилген милдеттерди турган жакын маанилеринин функциялары учурда ар кандай мааниде аргумент негизинде колдо болгон таблица маалыматтардын жардамы менен глобалдык жана локалдык интерполяция менен, экстраполяция милдеттери.

***Негизги сөздөр**: сызыктуу интерполирлөө, сплайн интерполирлөө, математикалык моделдөө, маалымат түздөө, сандык талдоо, эксперименталдык маалыматтар, натыйжалар.*

В представленной работе рассмотрены задачи интерполяции таблично заданной функции, сглаживания случайно сформированных данных, связанных с конкретными проблемами и вопросами прикладного характера. Эти задачи решены с использованием интегрированной программной системы для математических расчетов из класса систем автоматизированного проектирования, пакета MathCAD, с представлением выходных данных в виде графической информации. Математические пакеты, представляющие собой удобные и многофункциональные прикладные программы, помогают реализовывать численные эксперименты, решать большое количество аппроксимационных задач. Неоспоримые функциональные возможности пакета MathCAD предоставляют инструменты для математического моделирования широкого круга специализированных задач нахождения приближенных значений функции при любом значении аргумента на основе имеющихся табличных данных с помощью глобальной и локальной интерполяции, задач экстраполяции.

***Ключевые слова:** линейная интерполяция, сплайн-интерполяция, математическое моделирование, сглаживание данных, численный анализ, экспериментальные данные, результаты.*

In this paper, the problems of interpolation of a table-given function, smoothing of randomly generated data associated with specific problems and questions of an applied nature are considered. These problems are solved using an integrated software system for mathematical calculations from the class of computer-aided design systems, the MathCAD package, with the presentation of the output data in the form of graphical information. Mathematical packages, which are convenient and multifunctional applications, help to implement numerical experiments, solve a large number of approximation problems. The indisputable functionality of the MathCAD package provides tools for mathematical modeling of a wide range of specialized problems of finding approximate values of a function at any value of an argument on the basis of available tabular data using global and local interpolation, extrapolation problems.

***Key words:** linear interpolation, spline interpolation, mathematical modeling, data smoothing, numerical analysis, experimental data, results.*

The need to interpolate functions is mainly due to two reasons:

1. The function $f(x)$ has a complex analytical description that causes certain difficulties in its use (for example, $f(x)$ is a special function: gamma function, elliptic function, etc.).

DOI:10.26104/IVK.2019.45.557

2. The analytical description of the function $f(x)$ is unknown, i.e. $f(x)$ is given in a table. Thus, it is necessary to have the analytical description approximately representing $f(x)$ (for example, for calculation: values of $f(x)$ in arbitrary points, definition of integrals and derivatives of $f(x)$, etc.).

The MathCAD package contains tools for solving problems of interpolation of a tabular given function [1, 2].

Interpolation uses the values of some function given in a series of points to predict the values of the function between them. In MathCAD, you can connect data points with straight lines (linear interpolation) or connect them with polynomial segments (spline interpolation) [3, 4]. Interpolation functions define a curve that passes exactly through specified points. Because of this, the result is very sensitive to data errors.

In linear interpolation, MathCAD connects existing data points with straight lines. This is done by the function $linterp\,(vx, vy, x)$.

The function *linterp* uses data vectors vx and vy, to return an interpolated value y corresponding to the third argument x. The arguments vx and vy must be vectors of the same length. The vector vx must contain real values in ascending order.

This function connects data points with line segments, thus creating a polyline. The interpolated value for a particular is the ordinate of the corresponding polyline point.

Spline interpolation gives much better results. Spline interpolation allows you to draw a curve through a set of points such that the first and second derivatives of the curve are continuous at each point. This curve is formed by creating a series of cubic polynomials passing through sets of three adjacent points. The cubic polynomials then dock with each other to form a single curve.

For linear interpolation of a function specified in tabular form, you must perform the following steps:

1) Create vectors vx and vy containing the given table values of the argument x and function y. The elements vx must be arranged in ascending order.

2) Calculate the vector $vs(vx) = linterp\,(vx, vy, vx)$.

3) Build graphs of functions: given in tabular form $vy(vx)$ and calculated by linear interpolation $vs(vx)$.

To spline interpolate a function defined in tabular form, you must do the following:

1) Create vectors vx and vy containing the given table values of the argument x and function y. The elements vx must be arranged in ascending order.

2) Calculate the vectors of the second derivatives of the interpolation curve at the points under consideration:

$vs_1 = lspline(vx, vy)$ – for linear spline interpolation,

$vs_2 = pspline(vx, vy)$ – for parabolic spline interpolation,

$vs_3 = cspline(vx, vy)$ – for cubic spline interpolation.

3) Calculate spline interpolation vectors:

$vk_1(vx) = interp(vs_1, vx, vy, vx)$ – for linear spline interpolation,

$vk_2(vx) = interp(vs_2, vx, vy, vx)$ – for parabolic spline interpolation,

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$vk_3(vx) = interp(vs_3, vx, vy, vx)$ – for cubic spline interpolation.

4) Build graphs of functions: given in tabular form $vy(vx)$ and calculated by spline interpolation $vk_1(vx)$, $vk_2(vx)$, $vk_3(vx)$.

Task number 1

Perform linear and spline interpolation of the function specified in the table view.

x_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
y_i	0	0.1	0.2	0.3	0.47	0.5	0.61	0.7	0.8	0.9

Plot graphs of the original function as well as interpolation results. To compare the results.
Solving the problem of linear interpolation of the function:

1) Let's create vectors of the given table values of argument and function (Fig. 1):

$$VX := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \end{pmatrix} \quad VY := \begin{pmatrix} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.47 \\ 0.5 \\ 0.61 \\ 0.7 \\ 0.8 \\ 0.9 \end{pmatrix}$$

Fig. 1. Tables of values of argument VX and function VY

2) Calculate the vector of linear interpolation:

$$A(VX) := linterp(VX, VY, VX)$$

3) Construct graphs of functions: given in tabular form $VY(VX)$ and calculated by linear interpolation $A(VX)$ (Fig. 2).

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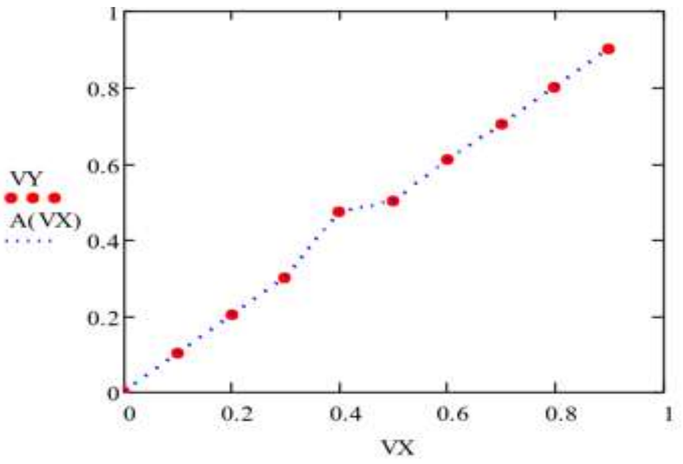

Fig. 2. Graphs of given $VY(VX)$ and computed functions $A(VX)$

Conclusion: from the analysis of the constructed graphs, it can be concluded that the graph of the linear function completely coincides with the graph of the original function and is a set of segments connecting the points of the original data. However, interpolation can not only connect the data points with lines, but also calculate the corresponding values that can be used to predict the value of the dependence at those points where it has not been measured by experience. Thus, linear interpolation gives an acceptable result for the given source data.

The solution of the problem of spline interpolation:

1) Let's create vectors of the given table values of argument and function.

2) Calculate the vectors of the second derivatives of the interpolation curve at the points under consideration:

$$VSb = lspline(VX, VY)$$

$$VSc = pspline(VX, VY)$$

$$VSd = cspline(VX, VY)$$

3) Calculate the spline interpolation vectors:

$$B(VX) = interp(VSb, VX, VY, VX)$$

$$C(VX) = interp(VSc, VX, VY, VX)$$

$$D(VX) = interp(VSd, VX, VY, VX)$$

4) Let's construct graphs of the functions given in table form $VY(VX)$ and calculated by spline interpolation $B(VX)$, $C(VX)$, $D(VX)$ (Fig. 3).

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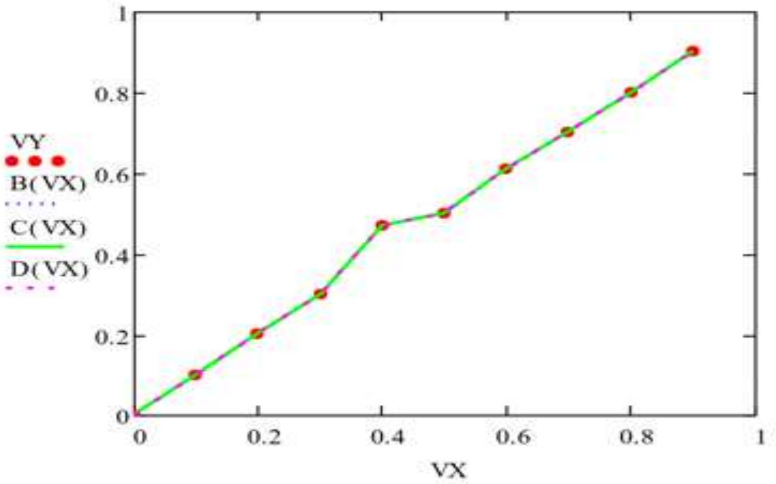

Fig. 3. Graphs of given $VY(VX)$ and computed functions $B(VX), C(VX), D(VX)$

Conclusion: from the analysis of the constructed graphs, it can be concluded that the spline interpolation graphs are smooth lines connecting the points of the original data. Within the set of values of all three lines is constructed by using polynomials of different types are the same. Thus, spline interpolation gives a good result for the given source data.

A very important task in the processing of experimental results is the smoothing of the obtained data. Smoothing always involves some way of local averaging the data, in which non-systematic components cancel each other out.

Smoothing involves using a set of values y and returning a new set of values y that is smoother than the original set. Unlike regression and interpolation, smoothing results in a new set of values y rather than a function that can evaluate values between given data points. Each element in the vector must have a value, because MathCAD assigns a value of 0 to any elements that are not explicitly defined.

To smooth the data, follow these steps:

1) Create vectors of initial values x and y. The elements of the vector x must be arranged in ascending order.

2) Perform data smoothing with built-in MathCAD functions:

$s_1 = medsmooth(y,n)$ – smoothing with moving median, $s_2 = ksmooth(x,y,b)$ – smoothing with Gaussian data core, $s_3 = supsmooth(x,y)$ – least squares smoothing.

Plot graphs: source data $y(x)$ and smoothing functions $s_1(x)$, $s_2(x)$, $s_3(x)$.

Task number 2

Using the *rnd* function, generate 50 random numbers from the range [0,5]. Perform data smoothing (using various built-in functions). Plot graphs of the original and smoothed functions. To compare the results.

Decision:

1) Let's create vectors of initial values from 50 random numbers from the range [0,5] (Fig. 4):

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$$i := 1..50$$

$$x_i := i$$

$$y_i := \text{rnd}(5)$$

x =

	0
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
11	11
12	12
13	13
14	14
15	...

y =

	0
0	0
1	2.183
2	2.889
3	3.143
4	2.521
5	3.479
6	0.95
7	0.892
8	2.287
9	0.488
10	0.472
11	4.657
12	4.473
13	1.137
14	2.054
15	...

Fig. 4. Vectors of initial values of 50 random numbers.

2) Perform data smoothing with built-in MathCAD functions:

$a = medsmooth(y, 11)$ – smoothing with moving median,

$b = ksmooth(x, y, 11)$ – smoothing with Gaussian data core,

$c = supsmooth(x, y)$ – smoothing by the method of least squares.

3) Let's build graphs: the initial data $y_i(x_i)$ and smoothed functions $a_i(x_i), b(x_i), c_i(x_i)$ (Fig. 5).

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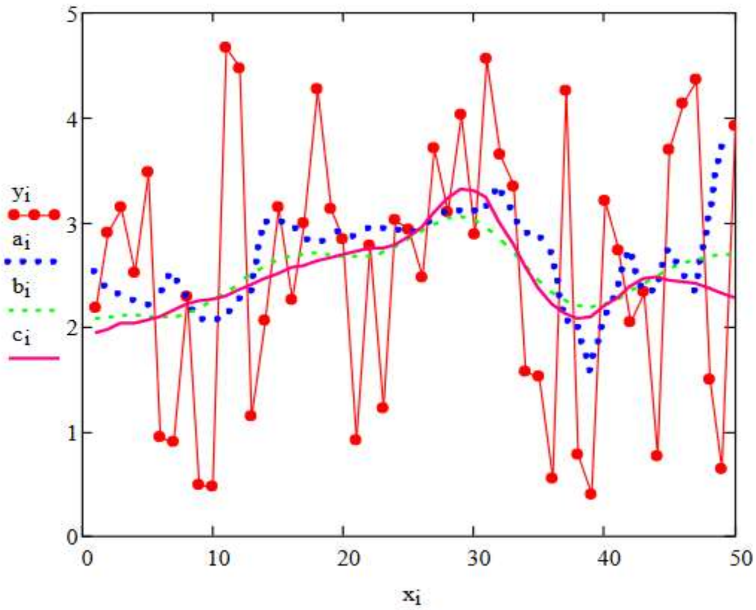

Fig. 5. Graphs of source data $y_i(x_i)$ and smoothed functions $a_i(x_i), b(x_i), c_i(x_i)$

Conclusion: from the analysis of the graphs, it can be concluded that the fundamental condition for success in smoothing the data is the correct choice of the algorithm depending on the nature of the original data. In addition, the results of smoothing data using the *medsmooth* and *ksmooth* functions are highly dependent on the selection of the smoothing window size. Thus, the best result of data smoothing was shown by the *supsmooth* function with adaptive selection of different smoothing bandwidth for different parts of the data.

Literature:

1. Дьяконов В.П. Mathcad 8-12 для студентов / В.П. Дьяконов. - М.: СОЛОН-ПРЕСС, 2005. - 589 с.
2. Kaliev I.A., Suyundukova A.K. Solving the problem of interpolation in the MathCAD mathematical package / I.A. Kaliev, A.K. Suyundukova // Теория. практика. инновации. - 2018. - №7. - С. 4-10.
3. Kaliev I.A., Suyundukova A.K. Solution of approximation problems of functions by means of MathCAD package / I.A. Kaliev, A.K. Suyundukova // Теория. практика. инновации. - 2018. - №7. - С.11-18.
4. Сабитова Г.С., Калиев И.А. Практикум по информационным технологиям. Часть 1: учебное пособие / Г.С. Сабитова, И.А. Калиев. - Стерлитамак: РИО Стерлитамакского филиала БашГУ. - 2015. - 127 с.